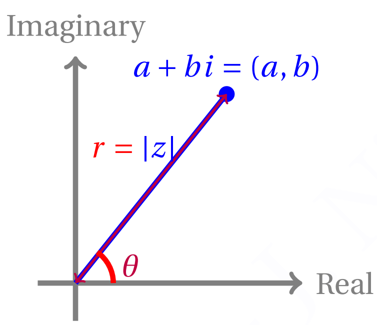
Worksheet Ten **MATH 104** Name:________________

WORK ON THIS ASSIGNMENT IN GROUP OF 2-4. TURN IN YOUR WORK INDIVIDUALLY IN CLASS. YOU CAN USE YOUR NOTES FOR THIS ASSIGNMENT.

8.5: Polar Form of Complex Numbers

- Expressing $z = a + bi$ in the Polar Coordinate:



- Polar Form of a Complex Number

Writing a complex number, $a + bi$, in polar form involves the following conversion formulas:

$a = |z|\cos(\theta)$, $b = |z|\sin(\theta)$.

Where absolute value of z is $|z| = r = \sqrt{a^2 + b^2}$ is a real number and θ is the angle made with polar axis in the polar form.

By direct substitution:

$$z = a + bi = (|z|\cos(\theta) + i|z|\sin(\theta)) = |z|\Big(\cos(\theta) + i\sin(\theta)\Big)$$

$|z|$ is called **modulus** and θ is called **argument** of z. In your book, we abbreviate $z = |z|cis(\theta)$. This is also called Euler Identity and is represented with $z = |z|e^{i\theta}$

Now, you can complete Problem 1.

Complex Operations Using the Polar Form:

- Multiplication:

If $z_1 = |z_1|\Big(\cos(\theta_1) + i\sin(\theta_1)\Big)$ and $z_2 = |z_2|\Big(\cos(\theta_2) + i\sin(\theta_2)\Big)$, then

$$z_1 \cdot z_2 = |z_1||z_2|\Big(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\Big)$$

Notice that the product calls for multiplying the moduli and adding the angles.

Now, you can complete Problem 2 Parts A-C.

- Division:

  If $z_1 = |z_1|\Big(\cos(\theta_1) + i\sin(\theta_1)\Big)$ and $z_2 = |z_2|\Big(\cos(\theta_2) + i\sin(\theta_2)\Big)$, then

  $\dfrac{z_1}{z_2} = \dfrac{|z_1|}{|z_2|}\Big(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\Big)$ where $z_2 \neq 0$.

  Notice that in finding the quotient of two complex numbers, you divide the moduli and subtract the arguments.

  Now, you can complete Problem 2 Part D and Problem 4.

- De Moivre's Theorem. (Finding an Integer Power of a Complex Number)

  If $z = |z|\Big(\cos(\theta) + i\sin(\theta)\Big)$ is a complex number, then

  $z^n = |z|^n\Big(\cos(n\theta) + i\sin(n\theta)\Big)$ where n is a positive integer.

  Now, you can complete Problem 2 Part E.

- Finding all nth root of a Complex Number

  If $z = |z|\Big(\cos(\theta) + i\sin(\theta)\Big)$ is a complex number, then

  $z^{\frac{1}{n}} = |z|^{\frac{1}{n}}\left[\cos\left(\dfrac{\theta}{n} + \dfrac{2k\pi}{n}\right) + i\sin\left(\dfrac{\theta}{n} + \dfrac{2k\pi}{n}\right)\right]$ for $k = 0, 1, 2, \ldots, n-1$.

  Now, you can complete Problem 3.

- Note on Calculations

  To be able to use any of the methods in operation with complex numbers, first find the polar coordinates of those complex numbers.

1. Express the following complex numbers in the polar form.

   (a) $5 + 5i$

   (b) $3 - 3i$

   (c) $1 + \sqrt{3}i$

   (d) $-1 + \sqrt{3}i$

2. Perform the following operations.

   (A) $\left[5\left(\cos\left(\dfrac{\pi}{3}\right) + i\sin\left(\dfrac{\pi}{3}\right)\right)\right] . \left[\sqrt{2}\left(\cos\left(\dfrac{\pi}{6}\right) + i\sin\left(\dfrac{\pi}{6}\right)\right)\right]$ (a.k.a. $5e^{\frac{\pi}{3}i}.\sqrt{2}e^{\frac{\pi}{6}i}$)

   (B) $\left[5\left(\cos\left(\dfrac{2\pi}{3}\right) + i\sin\left(\dfrac{2\pi}{3}\right)\right)\right] . \left[\sqrt{2}\left(\cos\left(\dfrac{\pi}{6}\right) - i\sin\left(\dfrac{\pi}{6}\right)\right)\right]$ (a.k.a. $5e^{\frac{2\pi}{3}i}.\sqrt{2}e^{\frac{-\pi}{6}i}$)

   (C) $\left[5\left(\cos\left(\dfrac{2\pi}{3}\right) + i\sin\left(\dfrac{2\pi}{3}\right)\right)\right] . \left[-\sqrt{2}\left(\cos\left(\dfrac{\pi}{6}\right) + i\sin\left(\dfrac{\pi}{6}\right)\right)\right]$

(D) $\dfrac{\left(\cos\left(\dfrac{2\pi}{3}\right)+i\sin\left(\dfrac{2\pi}{3}\right)\right)}{\sqrt{2}\left(\cos\left(\dfrac{\pi}{6}\right)+i\sin\left(\dfrac{\pi}{6}\right)\right)}$ (a.k.a. $\frac{e^{\frac{2\pi}{3}i}}{\sqrt{2}e^{\frac{\pi}{6}i}}$)

(E) $\left(\cos\left(\dfrac{2\pi}{3}\right)+i\sin\left(\dfrac{2\pi}{3}\right)\right)^7$ (a.k.a. $\left(e^{\frac{2\pi}{3}i}\right)^7$)

3. Perform the following operations to find the n^{th} roots of the give complex number.

(a) $\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)^{\frac{1}{2}}$ (All second roots of $\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$)

(b) $\left(128\left(\cos\left(\frac{7\pi}{3}\right) + i\sin\left(\frac{7\pi}{3}\right)\right)\right)^{\frac{1}{7}}$ (All 7^{th} roots of $\left(128\left(\cos\left(\frac{7\pi}{3}\right) + i\sin\left(\frac{7\pi}{3}\right)\right)\right)$.)

4. Find all complex solutions to the following equations.

(a) $x^3 = 125$

(b) $(x-1)^3 = 125$

INDIVIDUAL WORK

> UPLOAD TO CANVAS OR SUBMIT IN CLASS BEFORE DUE DATE. DISCUSSING THESE QUESTIONS IN YOUR GROUP IS ENCOURAGED BUT MAKE SURE YOU ARE TURNING IN YOUR OWN WORK.

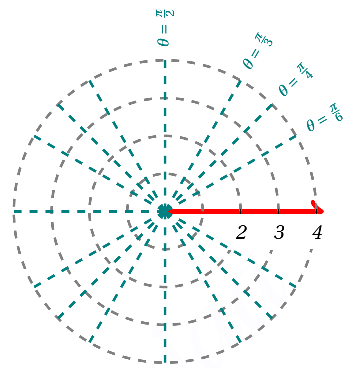
5. *(3 points) Use the Law of Sines to find all possible triangles with* $\angle C = 26^\circ$, $b = 12$ *and* $c = 9$.

6. **Polar conversions and Polar Form of Complex Numbers:**

   (A) *(1.5 points)* **Plot** *each of the following points in polar coordinates, and then* **convert** *them to points in rectangular coordinates.*

   $A\left(3\sqrt{2}, \frac{\pi}{4}\right) =$ __________

   $B\left(-2, -\frac{\pi}{3}\right) =$ __________



   (B) *(3 points) Convert each of the following polar coordinates equations to an equation in rectangular coordinates.*

   (i) $\theta = \dfrac{3\pi}{4}$

   (ii) $r = 6\cos(\theta)$.

   (C) *(0.5 points) Convert the complex number* $2 - 2i$ *to its polar form.*

   (D) *(1 point) Perform the following operation using the DeMoivre's law. Do NOT simplify.*

$$\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)^{99}$$